

THE EFFECTS OF THERMAL CONDUCTION ON THE HOT ACCRETION FLOWS

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ABSTRACT

Thermal conduction has been suggested as a possible mechanism by which sufficient extra heating is provided in radiation-dominated accretion flows. We consider the extreme case in which the generated energy due to the viscosity and the energy transported by a saturated form of thermal conduction are balanced by the advection cooling. For the steady-state structure of such accretion flows a set of self-similar solution are presented. Based on these solutions while the radial and the rotational velocities are both sub-Keplerian, increasing the level of thermal conduction has the effects of decreasing the rotational velocity, but increasing the radial velocity. Conduction provides extra heating and the temperature of the gas increases with thermal conduction.

Subject headings: accretion, accretion disks - hydrodynamics

1. INTRODUCTION

The thin accretion disk model describes flows in which the viscous heating of the gas radiates out of the system immediately after generation (Shakura & Sunyaev 1973). However, another kind of accretion has been studied during recent years where radiative energy losses are small so that most of the energy is advected with the gas. These Advection-Dominated Accretion Flows (ADAF) occur in two regimes depending on the mass accretion rate and the optical depth. At very high mass accretion rate, the optical depth becomes very high and the radiation can be trapped in the gas. This type of accretion which is known under the name ‘slim accretion disk’ has been studied in detail by Abramowicz et al. (1988). But when the accretion rate is very small and the optical depth is very low, we may have another type of accretion (Narayan & Yi 1994; Abramowitz et al. 1995; Chen 1995). However, numerical simulations of radiatively inefficient accretion flows revealed that low viscosity flows are convectively unstable and convection strongly influences the global properties of the flow (e.g., Igumenshchev, Abramowicz, & Narayan 2000). Thus, another type of accretion flows has been proposed, in which convection plays as a dominant mechanism of transporting angular momentum and the local released viscous energy (e.g., Narayan, Igumenshchev, & Abramowicz 2000).

This diversity of models tells us that modeling the hot accretion flows is a challenging and controversial problem. We think, one of the largely neglected physical ingredient in this field, is *thermal conduction*. But a few authors tried to study the role of “turbulent” heat transport in ADAF-like flows (Honma 1996; Manmoto et al. 2000). Since thermal conduction acts to oppose the formation of the temperature gradient that causes it, one might expect that the temperature and density profiles for accretion flows in which thermal conduction plays a significant role to appear different compared to those flows in which thermal conduction is less effective. Just recently, Menou (2005) studied the effect of saturated thermal conduction on optically thin ADAFs using an extension of self-similar solutions of Narayan & Yi (2004). His solutions suggest

that thermal conduction may be an important physical factor to understand hot accretion onto dim accreting black holes.

The aim of the work presented here is to investigate whether thermal conduction can affect the dynamics of an optically *thick* radiation-dominated accretion flow. In §2, we present a height-integrated set of equations of the model. Self-similar solutions are investigated in §3. The paper concludes with a summary of the results in §4.

2. FORMULATION

In order to implement thermal conductivity correctly it is essential to know whether the mean free path is less than (or comparable to) the scale length of the temperature gradient. For electron mean free path which are greater than the scale length of the temperature gradient the thermal conductivity is said to ‘saturate’ and the heat flux approaches a limiting value (Cowie & McKee 1977). But when the mean free paths are much less than the temperature gradient the heat flux depends on the coefficient of thermal conductivity and the temperature gradient. Generally, thermal conduction transfers heat so as to oppose the temperature gradient which causes the transfer. Menou (2005) discussed hot accretion likely proceed under weakly-collisional conditions in these systems. Thus, a saturated form of “microscopic” thermal conduction is physically well-motivated, as we apply in this study. However, one of the primary problems for studying the effects of thermal conduction in plasmas is the unknown value of the thermal conductivity.

We consider an accretion disk that is axisymmetric and geometrically thin, i.e. $h/r < 1$. Here r and h are, respectively, the disk radius and the half-thickness. Our model generalizes the usual slim disks around a neutron star or a black hole (e.g., Muchotrzeb & Paczyński 1982; Matsumoto et al. 1984; Abramowicz et al. 1988) by including the effect of thermal conduction. The disk is supposed to be turbulent and possesses an effective turbulent viscosity. We assume the generated energy due to viscous dissipation and the heat conducted into the volume is concerned are balanced by the advection cooling. Consider stationary height-integrated equations described an accre-

tion flow onto a central object of mass M_* . In absence of mass outflows, the continuity equation reads

$$\dot{M} = -2\pi r v_r \Sigma, \quad (1)$$

where \dot{M} is the accretion rate, v_r is the accretion velocity (and $v_r < 0$) and $\Sigma = 2h\rho$ is the surface density at a cylindrical radius. Also, ρ is the midplane density of the disk.

The equation of motion in the radial direction is

$$\Sigma v_r \frac{dv_r}{dr} - \Sigma \frac{v_\varphi^2}{r} = -\frac{dP}{dr} - \Sigma \frac{GM_*}{r^2}, \quad (2)$$

where $P = 2hp$ is the integrated disk pressure written, in a form compatible with radiation-dominated flow, as $P \simeq 2haT^4/3$, where a is black body constant and T denotes the midplane temperature of the disk. Similarly, integration over z of the azimuthal equation of motion gives

$$r\Sigma v_r \frac{d}{dr}(rv_\varphi) = \frac{d}{dr}[r^3\nu\Sigma \frac{d}{dr}(\frac{v_\varphi}{r})], \quad (3)$$

where ν is a kinematic viscosity coefficient and we assume

$$\nu = \alpha c_s h, \quad (4)$$

where α is a constant less than unity (Shakura & Sunyaev 1973). Moreover, $h = (c_s/v_K)r$, where c_s and v_K are sound speed and the Keplerian velocity, respectively: $c_s = \sqrt{P/\Sigma}$ and $v_K = \sqrt{GM_*/r}$.

Now, we can write the energy equation considering the cooling and the heating processes of the accretion flow. The locally released energy due to viscous dissipation, Q_{vis} , and the energy transported by thermal conduction, Q_{cond} , are balanced by the advection cooling, Q_{adv} . Thus,

$$Q_{\text{adv}} = Q_{\text{cond}} + Q_{\text{vis}}. \quad (5)$$

The advection cooling reads (Abramowicz et al. 1998)

$$Q_{\text{adv}} = \frac{\dot{M}}{2\pi r^2} \frac{p}{\rho} \left[\frac{4-3\beta}{\Gamma_3-1} \frac{d \ln T}{d \ln r} + (4-3\beta) \frac{d \ln \rho}{d \ln r} \right], \quad (6)$$

where β is the ratio of the gas to the total pressure and

$$\Gamma_3 = 1 + \frac{(4-3\beta)(\gamma-1)}{\beta+12(\gamma-1)(\beta-1)}. \quad (7)$$

Since we are considering the extreme case of radiation-dominated accretion flow, we have $\beta = 0$ and so $\Gamma_3 = 2/3$. Also, we have

$$Q_{\text{vis}} = \nu \Sigma r^2 \left[\frac{d}{dr} \left(\frac{v_\varphi}{r} \right) \right]^2, \quad (8)$$

and (Cowie & McKee 1977)

$$Q_{\text{cond}} = -\frac{2h}{r^2} \frac{d}{dr}(r^2 F_s) \quad (9)$$

where as we have already mentioned $F_s = 5\Phi_s \rho c_s^3$ is the saturated conduction flux on the direction of the temperature gradient ($\Phi_s < 1$).

3. SELF-SIMILAR SOLUTIONS

The self-similar solution is not able to describe the global behaviour of the accretion flow, because no boundary condition has been taken into account. However, as long as we are not interested in the the behaviour of the flow at the boundaries, such solution describes correctly the true solution asymptotically at large radii. We assume that each physical quantity can be expressed as a power

law of the radial distance, i.e. r^ν , where the power index ν is determined for each physical quantity self-consistently. The solutions are

$$\Sigma(r) = a \Sigma_0 \left(\frac{r}{r_0} \right)^{-1/2}, \quad (10)$$

$$v_\varphi(r) = b \sqrt{\frac{GM_*}{r_0}} \left(\frac{r}{r_0} \right)^{-1/2}, \quad (11)$$

$$v_r(r) = -c \sqrt{\frac{GM_*}{r_0}} \left(\frac{r}{r_0} \right)^{-1/2}, \quad (12)$$

$$P(r) = d \frac{\Sigma_0 GM_*}{r_0} \left(\frac{r}{r_0} \right)^{-3/2}, \quad (13)$$

$$h(R) = f r_0 \left(\frac{r}{r_0} \right), \quad (14)$$

where Σ_0 and r_0 provide convenient units with which the equations can be written in the non-dimensional form. Thus, we obtain the following system of dimensionless equations, to be solved for a, b, c, d and f :

$$ac = \dot{m}, \quad (15)$$

$$-\frac{1}{2}ac^2 - ab^2 = \frac{3}{2}d - a, \quad (16)$$

$$-\frac{1}{2}abc = -\frac{3\alpha}{4} \sqrt{\frac{d}{a}} f ab, \quad (17)$$

$$af^2 - 2d = 0, \quad (18)$$

$$\frac{3}{2} \dot{m} \frac{d}{a} = \frac{9}{4} \alpha \sqrt{\frac{d}{a}} f b^2 a + 5\Phi_s d \sqrt{\frac{d}{a}}, \quad (19)$$

where $\dot{m} = \dot{M}/(2\pi\Sigma_0\sqrt{GM_*r_0})$ is nondimensional mass accretion rate. After some algebraic manipulations we find

$$a = \left(\frac{2\sqrt{2}}{3\alpha} \dot{m} \right) \frac{1}{f^2}, \quad (20)$$

$$b = \sqrt{1 - \frac{3}{4}f^2 - \frac{9}{16}\alpha^2 f^4}, \quad (21)$$

$$c = \left(\frac{3\alpha}{2\sqrt{2}} \right) f^2, \quad (22)$$

$$d = \frac{\sqrt{2}}{3\alpha} \dot{m}, \quad (23)$$

and f is obtained from a forth order algebraic equation:

$$9\alpha^2 f^4 + 20f^2 - \frac{160\Phi_s}{9\alpha} f - 16 = 0. \quad (24)$$

We can solve this algebraic equation numerically and clearly only the real root which corresponds to positive b^2 is a physical solution.

If we neglect the thermal conduction (i.e., $\Phi_s = 0$), the above equation reduces to a second order equation which can be solved analytically (Wang & Zhou 1999; Shadmehri & Khajenabi 2005). In this case, the typical behaviour of the solutions can be summarized as follows (Shadmehri & Khajenabi 2005): (1) The surface density increases with the accretion rate, and decreases with viscosity coefficient α ; (2) But the radial velocity is directly proportional to α ; (3) The gas rotates with sub-Keplerian angular velocity, more or less independent of the coefficient α ; and (4) the opening angle of the disk is fixed, independent of α and \dot{m} .

Note that Shadmehri & Khajenabi (2005) applied a diffusive prescription for the viscous stress tensor in their analysis of the self-similar solution of optically thick advection-dominated flows, but Wang & Zhou (1999) applied the αp prescription. However, the typical behaviours of the solutions are the same, irrespective of some differences in the coefficients.

The equations (20), (21), (22), (23) together with equation (24) describes self-similar behaviour of the optically thick advection dominated accretion disk with saturated thermal conduction. The algebraic equation (24) shows that the variable f which determines the opening angle of the disk depends only on the α and Φ_s . This behaviour is similar to the case without thermal conduction. Thus, one can deduce that the surface density and the pressure are directly proportional to the mass accretion rate according to the equations (20) and (23). But the rotational and the radial velocities are both independent of the mass accretion rate (see equations (21) and (22)). Also, we can say that the rotational velocity is sub-Keplerian according to the equation (21).

The dependence of the properties of the rotational and the radial velocities on α and Φ_s is illustrated in Figure 1. We consider three values of the viscosity parameter, $\alpha = 0.2, 0.02$ and 0.002 for which the velocities are plotted as a function of Φ_s . The solid and dashed lines represent the ratios v_ϕ/v_K and v_r/v_K , respectively. While the solution without thermal conduction are recovered at small Φ_s values, we can see significant deviation as Φ_s increases. Clearly, the rotational velocity decreases with the magnitude of conduction parameter. But for the radial velocity the behaviour is different, i.e. the velocity increases as Φ_s increases. For a fixed viscosity parameter α , the solution reaches the non-rotating limit at a specific value of Φ_s which we denotes by Φ_s^c . If we extend our solution to $\Phi_s > \Phi_s^c$, the equation (21) gives negative b^2 which is clearly unacceptable. The critical value Φ_s^c can be calculated easily:

$$\Phi_s^c = \frac{9}{10\sqrt{6}} \sqrt{\sqrt{4\alpha^2 + 1} - 1}. \quad (25)$$

The above equation shows the critical magnitude of conduction parameter for which the solution tends to the non-rotating limit depends only on the viscosity coefficient so that as α increases, the breakdown of the solutions occurs at larger values of Φ_s . But as Menou (2005) mentioned in his analysis of optically thin ADAFs with saturated con-

duction, we think this behaviour is a simple pathological feature of 1D height-integrated equations we solved.

Solutions of equation (24) show that for a fixed α , the variable f increases with conduction parameter Φ_s . Since the gas temperature varies directly in proportion to f , we can say as the level of thermal conduction is increased the temperature of flow in comparison to solutions without thermal conduction increases. In fact, thermal conduction behaves as an extra heating source. Also, equation (20) shows the surface density decreases as Φ_s increases. We can compare the relative values of advection cooling, viscous heating and the transported energy by conduction using our self-similar solutions. One can simply show that $Q_{\text{vis}}/Q_{\text{adv}} = 2(b/f)^2$ and $Q_{\text{cond}}/Q_{\text{adv}} = 20\Phi_s/(9\alpha f)$. While both these ratios don't depend on the mass accretion rate, the viscosity parameter α and the thermal conduction parameter Φ_s determine the relative importance of the various physical processes in the system. Viscous energy dissipation decreases as Φ_s increases, but the contribution of the energy transported by the conduction becomes more significant.

4. CONCLUSIONS

In this paper we have studied an optically thick ADAFs with thermal conduction. Considering the weakly-collisional nature of hot accretion flows, a saturated form of thermal conduction has been used as a possible mechanism of transporting of energy. Although our self-similar solutions have their own limitations, they can illuminate some possible effects of the saturated thermal conduction on radiation-dominated accretion flows. We can summarize the main features of our solutions: (1) The surface density is in proportion to the mass accretion rate and it increases as the level of the thermal conduction is increased. (2) The temperature of the gas increases with thermal conduction parameter and the conduction term in the energy equation acts as an extra source of heating. (3) The angular velocity is sub-Keplerian, but it decreases with increasing thermal conduction parameter so that it tends to a non-rotating limit. (4) However, the radial velocity increases with thermal conduction parameter. Considering these significant effects of the thermal conduction, global transonic solutions representing optically thick (or thin) ADAFs with thermal conduction would be an interesting topic for future studies and our results are a step toward this goal.

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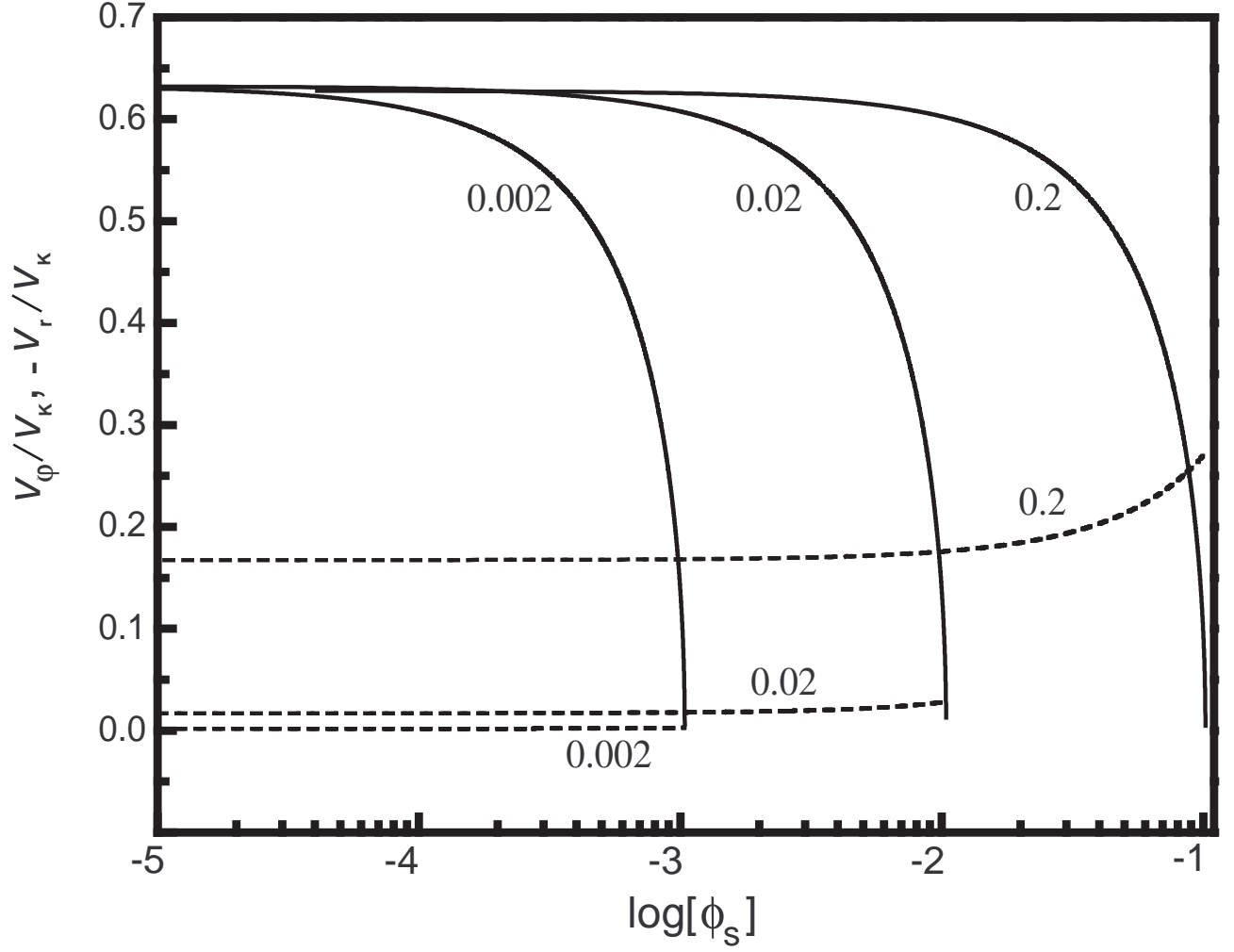


FIG. 1.— Profile of the rotation velocity (v_ϕ/v_K , solid lines) and the radial velocity (v_r/v_K , dashed lines) as a function of the saturation constant, Φ_s . In order to make easier comparison the rotational and the radial velocities are scaled by the Keplerian velocity, v_K . Each curve is labeled by corresponding viscosity coefficient, α .